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TRANSONIC RAREFACTION FLOW IN THE NEIGHBORHOOD OF A CONVEX CORNER*

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A new solution of the problem of perfect gas flow over a corner is considered in the case when a sonic line issues from the corner point, and a mixed flow is formed. It satisfies the Fal'kovich—Kármán equation, belongs to its class of self-similar solutions /1/, and obtains when the self-similarity parameter $n = 2$.

The obtained solution corresponds to the flow represented in Fig.1 in which perturbations from the supersonic part of the stream reach the corner point. The respective influence region is bounded by the limit characteristic C_0^-. Discontinuities of second derivatives of vector velocity components propagate along the limit characteristic C_0^+ to the region of flow. Beyond the limit characteristic C_0^+ the flow becomes a Prandtl—Meyer flow. Acceleration at the corner point is finite, and the sonic line is concave toward the oncoming stream. The flow constructed for $n = 2$ substantially differs form the Vaglio-Laurin flow which obtains for $n = {}^5/_4$, and was investigated in detail in /2—6/ and ,also, from the Vaglio-Laurin type which are defined by self-similar solutions of the Fal'kovich—Kármán equations for flow over corner points with subsonic generatrices of nonzero curvature /7/.

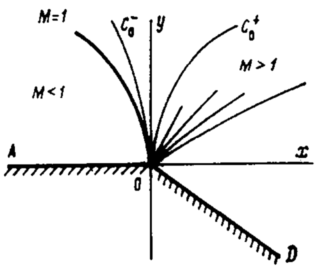

Fig.1

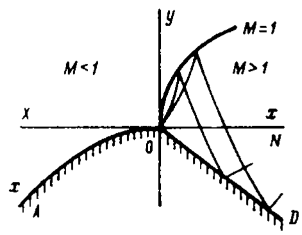

Fig.2

Transonic flow past convex walls defined, respectively, by the equations $y = 0$ and $y = cx$ $(c < 0)$ for $x \leqslant 0$ and $x \geqslant 0$ and the effect on the pattern of flow reaching the corner point along the limit characteristic C_0^- of discontinuity of first derivatives of the velocity vector components.

1. We consider the flow over the corner point O defined by the intersection of two smooth curves AO and OD (Figs.1,2).

We introduce the orthogonal system of coordinates (x, y) whose negative x-semiaxis coincides with AO and the positive, denoted in Fig.2 by ON, coincides with the tangent to AO at point O. The quantity y is measured along the external normal to AON. We define the curvature of the generatrix AO by the formula

$$\nu(x) = -3\frac{n-1}{n}B(n)(-x)^{2-3/n}[1+\varkappa(x)], \quad x \leqslant 0 \tag{1.1}$$

where $\varkappa(x) = o(1)$ as $x \to 0$. This condition is equivalent to that in the Cartesian system of coordinates (X, y) in which the X axis lies on the tangent to AO at point O, the equation of generatrix AO in the neighborhood of point O is of the form

$$y = -\frac{B(n)}{(4-3/n)}(-X)^{4-3/n}+\ldots \tag{1.2}$$

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Curvature (1.1) of the generatrix AO is $O(1)$ when $n \geqslant {}^3/_2$. If $B(n) \neq 0$ then when $n \in (1, {}^3/_2)$, the quantity $|\nu(x)| \to \infty$ as $x \to 0$. For certain values of n the coefficient $B(n)$ vanishes. These values of n are for $\nu(x) \equiv 0$, and correspond to flow over a corner. We denote by v_x and v_y projections of the velocity vector on axes x and y, respectively; and by a the speed of sound. As the characteristic values of all flow parameters we take their critical values. The thermodynamic variables are linked by the equation of state of perfect gas. In what follows all equations that define the flow are assumed to be dimensionless. They are of the form

$$a^2\nabla \mathbf{q} - \mathbf{q}\nabla \frac{\mathbf{q}^2}{2} = 0, \quad \mathbf{q} = (v_x, v_y) \tag{1.3}$$

$$a^2 = \frac{1+\gamma}{2}[1 - \mu^2(v_x^2 + v_y^2)], \quad \mu^2 = \frac{\gamma - 1}{\gamma + 1}, \quad \nabla \times \mathbf{q} = 0$$

The metric is defined by formulas

$$dl^2 = (1 - \sigma y)^2 dx^2 + dy^2, \quad \sigma(x) = \begin{cases} \nu(x), & x \leqslant 0 \\ 0, & x \geqslant 0 \end{cases}$$

We denote by G the neighborhood of point O where $|\sigma y| \ll 1$, and introduce the perturbation velocity $v_x = 1 + u$, $v_y = v$ $(|u| \ll 1, |v| \ll 1)$. It is now possible to simplify system (1.3) in region G, and represent it in the first approximation in the form

$$-(1+\gamma)u\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = -\sigma(x) \tag{1.4}$$

Carrying out the substitution

$$v = V - \int_0^x \sigma(x)\,dx$$

and introducing the potential φ of velocities (u, V), we obtain the Fal'kovich—Kármán equation

$$-(1+\gamma)\frac{\partial \varphi}{\partial x}\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0 \tag{1.5}$$

As in /2,5—7/, we seek a solution of Eq.(1.5) in region G in the class of self-similar functions that satisfy the following conditions:

a) when $y \to 0, x < 0$, the velocity component $v_y \to 0$, which is equivalent to

$$\frac{\partial \varphi}{\partial y} = V \to \int_0^x \sigma(x)\,dx = B(n)(-x)^{3-3/n} + \ldots$$

b) when $y \to 0, x > 0$, the flow becomes a Prandtl—Meyer flow.

When $n = {}^5/_4$, the problem has a solution which defines the flow over a convex corner, since $B({}^5/_4) = 0$; it was first obtained by Vaglio-Laurin /2/. In the Vaglio-Laurin flow (Fig.2) the acceleration at the corner point is infinite. Perturbations from the supersonic part of the flow do not reach it. From the sonic line which is convex toward the oncoming stream they propagate downstream along the characteristic. The effect of the subsonic generatrix AO can be taken into account using the following approximations to the Vaglio-Laurin solution. When $B(n) \neq 0$ and $1 < n < 2$, we also have solutions that define the flow over a corner point with generatrix AO of nonzero curvature whose effect is taken into account in the first approximation. We shall call them solutions of the Vaglio-Laurin type, since their properties are the same as those of the Vaglio-Laurin. The case of $n = {}^3/_2$ was considered in /7/.

In what follows the main efforts are related to the investigation of the problem for $n = 2$, which is a limit case in the sense that when $n \geqslant 2$ the Vaglio-Laurin type flow is not realized. A solution defining the flow over a corner when $n = 2$ is constructed below.

For arbitrary n the solvability of the problem with conditions a) and b) is conveniently investigated by the hodograph method. We denote by η the absolute value of velocity deviation from sonic and by θ the angle of the velocity vector inclination in the system of coordinates (X, y). The stream function ψ satisfies in the hodograph plane (η, θ) the Tricomi equation. We seek for it a solution of the form

$$\psi = \theta^{2j}\chi(j, \zeta), \quad \zeta = \frac{4\eta^3}{9\theta^2}$$

The corner point with a Prandtl—Meyer flow in its neighborhood but not in the hodograph plane is mapped onto the Γ_0^--characteristic $\theta = -(2\eta^{3/2})/3$. The solution which vanishes on that characteristic is of the form

$$\psi = \theta^{2j}(1-\zeta)^{2j+1/6} F\left(j+\frac{2}{3}, j+\frac{1}{6}, 2j+\frac{7}{6}, 1-\zeta\right), \tag{1.6}$$

$$j = \frac{1}{6(n-1)}$$

A solution of the Vaglio-Laurin type is an analytic continuation of solution (1.6) into the neighborhood of axis $\theta = 0$, $\eta < 0$ through axis $\eta = 0$, $\theta < 0$. When $n = 5/4$ stream function ψ vanishes when $\theta = 0$. In the case of $n = 3/2$ the derivation of solution is similar. However, if $n = 2$ is assumed, the analytic continuation of function (1.6) does not yield the solution $\psi = 0$.

2. To obtain a clearer picture of this situation and derive a solution of the problem with conditions a) and b) when $n = 2$, we turn to the Fal'kovich—Kármán equation (1.5). Its self-similar solutions and the equations which define the latter are of the form /8,9/ (γ is the specific heat ratio)

$$\varphi = y^{3n-2}\Phi(\xi), \quad u = \frac{\partial\varphi}{\partial x} = \lambda y^{3n-2} f(\xi), \quad V = \frac{\partial\varphi}{\partial y} = y^{3n-3} g(\xi), \quad f(\xi) = \frac{d\Phi}{d\xi}, \quad \xi = \lambda\frac{x}{y^n} \quad \lambda = (1+\gamma)^{-1/3}$$

$$\left(n^2\xi^2 - \frac{d\Phi}{d\xi}\right)\frac{d^2\Phi}{d\xi^2} - 5n(n-1)\xi\frac{d\Phi}{d\xi} + 3(n-1)(3n-2)\Phi = 0 \tag{2.1}$$

When $n = 2$, for functions f and g we have

$$(4\xi^2 - f)\frac{d^2f}{d\xi^2} - \left(\frac{df}{d\xi}\right)^2 - 2\xi\frac{df}{d\xi} + 2f = 0, \quad g = \frac{1}{3}\left[4\xi f + (f - 4\xi^2)\frac{df}{d\xi}\right] \tag{2.2}$$

Passing to phase variables /1,10/

$$f = \xi^2 F(\tau), \quad \frac{dF}{d\tau} = \Psi, \quad \tau = \ln|\xi|$$

we reduce the first of Eqs.(2.2) to the ordinary first order differential equation /10/

$$\frac{d\Psi}{dF} = \frac{-6F - 10\Psi + 6F^2 + 7\Psi F + \Psi^2}{(4-F)\Psi} \tag{2.3}$$

The relation between variables (f, ξ) and (F, Ψ) is defined by formulas

$$F = \xi^{-2} f(\xi), \quad \Psi = -2\xi^{-2} f + \xi^{-1}\frac{df}{d\xi}, \quad \frac{d\Psi}{dF} = \frac{1}{\Psi}\left[4\xi^{-2} f - 3\xi^{-1}\frac{df}{d\xi} + \frac{d^2f}{d\xi^2}\right] \tag{2.4}$$

Let us consider the behavior of integral curves of Eq.(2.3) in the phase plane (F, Ψ) (Fig. 3) and establish their relation with the solutions of Eqs.(2.2).

Equation (2.3) has four singular points A (0, 0), B (0, 1), C (4, −6), D (4, −12) in the finite part of the plane (F, Ψ) and three singular points E, G, Q which lie at infinity. Point A is a node and in the physical plane corresponds to the x axis. The behavior of integral curves in its neighborhood is defined by

$$\left(\frac{2}{n}F + \Psi\right) = C_A\left|\frac{3}{n}F + \Psi\right|^{3/2} + \ldots, \quad n = 2 \tag{2.5}$$

Point B is a saddle through which pass two integral curves. One of these represents the linear function $\Psi = 2 - 2F$ and in the point B neighborhood defines the Prandtl—Meyer flow. Point C is a dicritical node and corresponds to the limit characteristic C_0^+ issuing from the corner point, and point D is a saddle that corresponds to the limit characteristic C_0^- which enters the corner point.

The singular point G is a saddle which can be reached only along the integral curve $\Psi = -3F/2$ which also passes through point A and, as implied by (2.5), is one of the axes of node A. In the physical plane point G corresponds to the y axis on which sonic velocity is reached. The singular point E is an analytic node in whose neighborhood the behavior of integral curves can be represented in the form

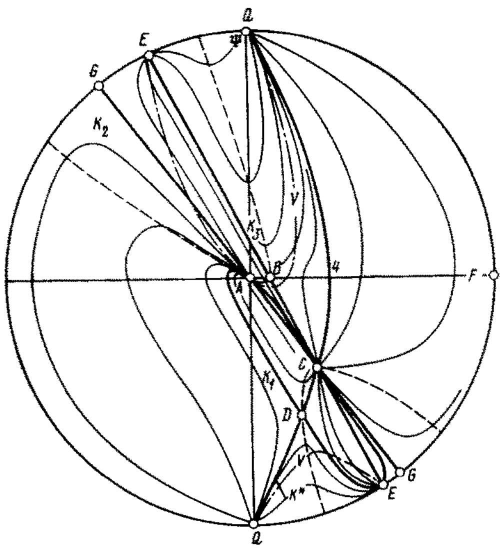
$$t = C_E|-2t + 2 + z|^2 + \ldots, \quad F = \frac{1}{t}, \quad \Psi = \frac{z}{t}, \quad t \to 0$$

One of the axes of node E is represented by the integral straight line $\Psi = 2 - 2F$ which, as indicated earlier, passes through the saddle B. In the physical plane point E corresponds to the y axis. Point Q is a node which is reached by moving along the limit line $|\Psi| \to \infty$, $F = 4$. At transition through point Q the quantity ξ reaches its extremum, which means that the plane is covered twice. At the approach to point Q the stream acceleration infinitely increases. Besides the indicated integrals $\Psi = -3F/2$, $\Psi = 2 - 2F$, there is one more integral of Eq.(2.3) obtained in the explicit form

$$\Psi = -(1 + 2F \pm \sqrt{1 + 2F}) \qquad (2.6)$$

The integral curve (2.6) issues from point A and, having passed through points D, E, C, returns to point A. It defines the analytic flow in a Laval nozzle /3, 10/. The line along which the velocity $v_y = 0$ is denoted by V.

As $|\xi| \to \infty$ the asymptotic behavior of solutions of Eq.(2.1) in the x axis neighborhood is defined by



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$$\Phi = A_0(\pm\xi)^{3-2/n} \pm \frac{(3n-2)^2(n-1)}{n^3} A_0^2(\pm\xi)^{3-4/n} + \qquad (2.7)$$

$$B_0(\pm\xi)^{3-3/n} \pm \frac{(n-1)(3n-2)(4n-5)}{2n^3} A_0B_0(\pm\xi)^{3-5/n} + \ldots$$

where A_0 and B_0 are aribtrary constants related to constant C_A in formula (2.5) by the formula

$$C_A = \frac{3(n-1)n}{(3n-2)^{3/2}} \frac{B_0}{|A_0|^{3/2}}$$

The case of $B_0 = 0, A_0 \neq 0$ $(C_A = 0)$ corresponds in the physical plane to the flow over a corner with a rectilinear generatrix, since when $B_0 = 0$, the quantities $\nu(x) = 0$ in (1.1) and $y = 0$ in (1.2), and $v_x \neq 0, v_y = 0$. The integral curve (2.5) that corresponds to this case will be called symmetric and denoted by K_1. In the case of $B_0 \neq 0$, $A_0 = 0$ $(C_A = \infty)$ the integral curve (2.5) will be called antisymmetric and denoted by K_2. The latter corresponds to flow past a wall (1.2) at sonic velocity at the boundary. The integral curve that passes through point B and corresponds to the Prandtl—Meyer flow will be denoted by K_3.

In the case of subsonic velocities on AO $(F < 0)$ one of the integral curves (2.5) reaches point G, and then, when $F < 0$, all remaining integral curves which lie between the latter and curve K_2 reach point E. One of them, in turn, reaches point B, unavoidably intersecting axis $F = 0$, which corresponds to passing through the speed of sound. In the physical plane to such motion corresponds the following path: moving from the negative semiaxis x we pass axis y, then the sonic line $\xi_{3B} > 0$ and, finally, obtain the Prandtl—Meyer flow. When $n = 5/4$ K_1 is such curve /2,3/, which corresponds to flow over a corner with rectilinear generatrix. When $n = 3/2$ the directrix of the corner has a constant curvature, and point B is reached by one of curves (2.5) with $C_A \in (0, \infty)$. When $n = 2$ the integral curve K_1 is transformed into curve (2.6), and curve $\Psi = 2 - 2F$ become curve K_3 which reaches point E when $C_E = \infty$, and to the integral curve K_2 corresponds curve $\Psi = -3F/2$ which, as previously indicated, reaches point G. This means that when $n = 2$, it is impossible to reach point A by continuous motion from point B on the upper part of curve K_3 $(\Psi > 0)$.

When $n = 2$ all integral curves, except curve K_3, are represented in the phase plane (F, Ψ) using the integrals

$$f = a\xi + \frac{a^2}{2} \pm \sqrt{\frac{1}{4}ab + b\xi}, \quad a, b = \text{const} \qquad (2.8)$$

Integrals (2.8) were used in /8,10/ for investigating flows in Laval nozzles with $b > 0$.

Note that functions F and Ψ are invariant with respect to the sign of ξ. In the neighborhood of point A solution (2.8) is the same as the expansion of $d\Phi/d\xi$ obtained by setting

$$a = 2A_0, \quad \sqrt{b} = \pm {}^3/_2 B_0 \ (\xi \to +\infty),$$
$$\sqrt{-b} = \pm {}^3/_2 B_0 \ (\xi \to -\infty)$$

Consider the neighborhood of point A. The integral curves lying above curve K_1 are defined by integrals (2.8) with the minus sign. For those of them that are included in the region between K_1 $(F > 0, \Psi < 0)$ and K_2 $(F < 0, \Psi > 0)$, (region 1) $a < 0$, $b < 0$ as $\xi \to -\infty$ and $a > 0$, $b > 0$ as $\xi \to +\infty$. The integral curves lying below curve K_1 are defined by integral (2.8) with the plus sign. In the region between K_1 $(F > 0, \Psi < 0)$ and K_2 $(F > 0, \Psi < 0)$ (region 2) we have $a < 0$, $b < 0$ as $\xi \to -\infty$ and $a > 0$, $b > 0$ as $\xi \to +\infty$. Curves K_1 and K_2 are obtained from (2.8) for $b = 0$ and $a = 0$, respectively. Curve K_3 is obtained by the direct integration of

$$f = \xi^2 + C, \quad C - \text{const} \tag{2.9}$$

In region $F < 0$ it is possible to reach point G that corresponds to the y axis from point A only by moving along K_2. The y axis is in that case the sonic line and, thus, also a characteristic. In the physical plane the flow defined by the integrals $f = -\sqrt{b}\xi$, $g = [(b/2) - 2\xi\sqrt{\xi b}]/3$, $\xi < 0, b < 0$ corresponds to curve K_2 $(F < 0)$. To extend this solution continuously to region $\xi > 0$ is impossible, even when, after having reached point G along curve K_2, one moves on it in the reverse direction to point A when $F < 0$. Function g becomes discontinuous on axis y. The jump from point G to point E implies discontinuity of velocities v_x, v_y. When the sonic line is defined by the equation $\xi = \xi_{3B} > 0$ (i.e. it is convex toward the oncoming stream), perturbations reach each of its points moving along characteristics C^+ and, then, carried away along C^- characteristics. When $n = 2$, these characteristics merge with the sonic line, and the flow becomes blocked. It is thus impossible to obtain a flow free of singularities (of the Vaglio-Laurin type) over a corner point with the generatrix AO (1.2), when $n = 2$ and $B(n) \neq 0$.

3. When $\Psi < -F$ $(F < 0)$, the integral curves issuing from point A reach either point Q or C, and only one curve (of integral (2.6)) reaches point D. It was shown in /10/ that integral is the unique symmetric solution K_1 which analytically passes the limit characteristic C_0^-. The integral curve (2.6) then passes through point E and reaches point C which in the physical plane issuing from the corner point corresponds to the limit characteristic C_0^+. Various solutions with discontinuities of derivatives can be joined along it. Integral (2.6) and the limit characteristics C_0^- and C_0^+ in terms of variables (ξ, f) are of the form

$$f = a\xi + \frac{a^2}{2}, \quad a > 0; \quad \xi_D = -\frac{a}{4}, \quad \xi_C = \frac{a}{2}$$

Consider the integral curves issuing from point C with $F \leqslant 4$, denoting by k the tangent of the inclination angle of these at C. Part of them reach point A, one reaches point B, and the remaining converge at Q. For the integral curves in region 1 we have $a_1 > 2\xi_C = a$, $k \in (-2, -{}^5/_3)$. As implied by (2.4) and (2.8), constants b_1, a_1, ξ_C, k are linked by the relations

$$b_1 = (a_1 - a)^2(a_1 + 2a), \quad k = -\frac{5}{3} - \frac{a_1 - a}{3(a_1 + 2a)} \tag{3.1}$$

When $a_1 = a$, the quantities $k = -{}^5/_3$, $b_1 = 0$ and the integral curve coincides with K_1. Fixing, b_1 and a we obtain from equalities (3.1) their homologous terms a_1 and k. The magnitude of discontinuity of the second derivative $d^2f/d\xi^2$ is determined by the value of k in (2.4). All integral curves issuing from point C and lying in region 1 intersect line V and, changing the sign of vertical velocity /component/ v_y to negative, reach point A. This case corresponds to a flow past a convex wall

$$y = 0, \ x \leqslant 0$$
$$y = -[4\sqrt{b_1}(\lambda x)^{5/2}]/15, \ x > 0 \tag{3.2}$$

For $k = -2$ equalities (3.1) loose their meaning. This is due to that integral (2.9) does not belong to the set of integrals (2.8). The transition from point C to point B means passing to the Prandtl—Meyer flow which in the physical plane corresponds to flow over a corner. The constant C in integral (2.9) obtained using formulas (2.4) is $3\xi_C^2$. The flow over a convex corner may be considered as the limit case of flow over a convex parabolic wall $y = 0$, $x \leqslant 0$, and $y \sim -\sqrt{b_1}x^{5/2}$, $x \geqslant 0$ as $b_1 \to \infty$.

For an arbitrary $n > 1$ the integral K_3 in the neighborhood of point B can be represented in the form

$$f = \xi^2[1 + C\xi^{-6/(n+1)} + \ldots], \quad \xi \to +\infty \tag{3.3}$$

where constant C is negative when $\Psi > 0$ and positive when $\Psi < 0$. This means that the velocity /component/ v_x of the flow over a convex corner with generatrix AO (1.2) and $n \in (1.2)$ of the Vaglio-Laurin type is always lower than the longitudinal velocity /component/ in the flow over a corner with $n = 2$. As a consequence, the pressure and density in the first case are higher than in the second. Hence the solution of the problem of flow over a convex corner in corner G is, when $n = 2$, of the form

$$\varphi = \begin{cases} \frac{a}{2}(\lambda x)^2 + \frac{a^2}{2}(\lambda x) y^2 + \frac{a^3}{24} y^4 \left(x < 0, y > 0; x > 0, y \geqslant \frac{a}{2\lambda}\sqrt{x}\right) \\ \frac{3}{4} a^2 (\lambda x) y^2 + \frac{1}{3}(\lambda x)^3 y^{-2} \left(x > 0, 0 < y < \frac{a}{2\lambda}\sqrt{x}, a > 0\right) \end{cases}$$

We fix the constant b_1 and consider the subsonic flow past a convex wall. The dependence of a_1 on a is determined using the cubic equation (3.1) with the condition that $a_1 > a$. We have

$$a_1 = 2a\cos\left(\frac{\pi - \theta}{3}\right), \quad \cos\theta = 1 - \frac{b_1}{2a^3}$$

$$0 < \theta < \frac{\pi}{2}, \quad -\frac{5}{3} < k < -\sqrt{3}, \quad a > (b_1/2)^{1/3}$$

$$a_1 = 2a\cos\frac{\theta}{3}, \quad \cos\theta = \frac{b_1}{2a^3} - 1$$

$$0 < \theta < \pi/2, \quad -\sqrt{3} \leqslant k \leqslant -{}^7/_4, \quad (b_1/4)^{1/3} \leqslant a \leqslant (b_1/2)^{1/3}$$

$$a_1 = 2a\,\mathrm{ch}\frac{\theta}{3}, \quad \mathrm{ch}\,\theta = \frac{b_1}{2a^3} - 1$$

$$0 < \theta < \infty, \quad -7/4 < k < -2, \quad 0 < a < (b_1/4)^{1/3}$$

The second derivatives of functions f and g are discontinuous on the limit characteristic C_0^+. On approaching it from the left they are equal zero, and behind it they are determined by formulas

$$\left[\frac{d^2 f}{d\xi^2}\right]_{\xi=\xi_C} = -6\left(k + \frac{5}{3}\right), \quad \left[\frac{d^2 g}{d\xi^2}\right]_{\xi=\xi_C} = 12\xi_C\left(k + \frac{5}{3}\right), \quad \left[\frac{d^3 g}{d\xi^3}\right]_{\xi=\xi_C} = 12\left(k + \frac{5}{3}\right) \tag{3.4}$$

Formulas (3.4) imply that the second derivatives have their maximum jump in the flow over a corner at crossing the characteristic C_0^+. As $\xi_C \to 0$, the jump of the second derivative of function g approaches zero. When ξ_C vanishes, the oncoming stream velocity becomes sonic and the third derivative of g suffers a discontinuity. It will be seen from (2.9) that $f_1 = \xi^2$ when $\xi_C = 0$, which means that then the sonic stream merges along the characteristic with the Prandtl —Meyer flow. If we pass now from the derived solution (u, v) to the function of the potential, the local solution $\varphi(x, y)$ of the Fal'kovich—Kármán equation (1.5) will belong to $C^2(G)$ and satisfy conditions a) and b). The selection of solution is determined by the over-all problem.

4. Let us now assume that along the characteristic C_0^- $\xi_D = -a/4$, and that singularities in the form of discontinuities of first derivatives of velocity components, reach the corner point. This problem was considered in detail in /10/ in relation to the Laval nozzles. In this case the integral curve K_1 having reached point D passes with a jump to point C in the phase plane. The further flow pattern depends on the inclination of the integral curve k_D issuing from point C. At some values of k_D shock-free flows are realized, at others, only flows with shock waves obtained. The jump of the second derivative of function f on characteristic C_0^- is $[d^2f/d\xi^2]_{\xi=\xi_D} = -6\,(k_D + {}^5/_3)$.

If $k_D \in (-\infty, -2)$, then

$$f = \alpha\xi + \frac{\alpha^2}{2} - \sqrt{1/4\alpha\beta + \beta\xi}, \quad \alpha > 0, \ \beta > 0, \ \xi \in [\xi_D, \xi_C] \tag{4.1}$$

$$\alpha = \frac{a}{2}\,\frac{2k_D + 3}{k_D + 2}, \quad \sqrt{|\beta|} = -24\left(k_D + \frac{5}{3}\right)\left[-\frac{a}{8(k_D + 2)}\right]^{3/2}$$

The integral curves (4.1) issue from point C, reach point E (the y axis) and return to point C, which corresponds to reaching the C_0^+ characteristic. Further, the integral curve issuing from C reaches along K_3 point B. On characteristic C_0^+ the second derivative becomes discontinuous with the jump

$$\left[\frac{d^2 f}{d\xi^2}\right]_{\xi=\xi_C} = 6(k_C + 2) > 0, \quad k_C \in (-{}^5/_3, -2) \tag{4.2}$$

When $k_D = -2$ the motion takes place from point C along curve K_3 to point E, then back from point E to point C along K_3, and then to point B. The vertical velocity /component/ V_y on axis y changes its sign from the negative to positive. The constant C in (2.9) is equal $3\xi_D^2$, and the jump of the second derivative on characteristic C_0^+ vanishes.

When $k_D \in (-2, -5/3)$, motion takes place along integral curves (4.1), but $\alpha < 0$, $\beta < 0$. Along the characteristic C_0^+ second derivatives become discontinuous with shock (4.2), but $k_C \in (-\infty, -2)$. The case of $k_D = -5/3$ is a limit one. Motion takes place along the integral curve K_1 to point D, which corresponds to reaching the characteristic C_0^+. Now, to reach point B it is necessary to pass with a jump to point C. Thus, when $k_D = -5/3$ the singularity is reflected from the corner point in the form of discontinuity of first derivatives. The constant C in (2.9) and the quantities k_C, a, and k_D are related by formulas

$$4\xi_C^2 = \alpha\xi_C + {}^1/_2\alpha^2 - \sqrt{{}^1/_4\alpha\beta + \beta\xi_C}, \quad \xi_C = \sqrt{C/3}$$
$$^1/_4\beta^2\,({}^1/_4\alpha\beta + \beta\xi_C)^{-1/2} = -6\,(k_C + {}^5/_3)$$

The relations between constants a and C for which a shock-less transition from point A to point B is effected are of the form

$$\frac{3}{64} \leqslant \frac{C}{a^2} \leqslant \frac{3}{4}$$

If now $k_D > -5/3$, the integral curves issuing from point C lie between K_1 and K_2. After passing through point E all of them reach point Q. This means that the derivative $df/d\xi$ becomes infinite for some ξ greater than zero. However, a motion of gas with infinite acceleration is not realizable. Either a shock wave is generated in it, or the whole flow disintegrates. The state of gas in the phase plane ahead and behind the shock wave are related by formulas /10/

$$F_1 + F_2 = 8, \ \Psi_1 + \Psi_2 = -36, \ 1 < F_2 < 4$$

Using these and integral (2.9) we obtain the relation between the quantities F_1 and Ψ_1

$$\Psi_1 = -2F_1 - 22, \ 4 < F_1 < 7$$

We denote the obtained set by K^*, and the slope of the integral curve issuing from point C and passing through point $F = 7$ by $\Psi = -36, -k_D^*$. Now, when $k_D \in (-5/3, k_D^*)$, it is possible to construct the flow over an obtuse corner, by introducing in the flow the shock wave $\xi = \xi_S > 0$. The variation range of constants C and a are determined by the inequality $0 < C/a^2 < 3/64$.

An example of flow over a corner with discontinuities of velocity component derivatives along the characteristic issuing from the corner point was given in /11/, where a vortex flow in the neighborhood of the corner was considered in the case of $n = 3/2$. We point out in concluding that when $n = 2, 3, 11$, asymptotic types of flow in Laval nozzles are realized /10/. Hence it is possible to construct solutions that define the flow over an obtuse corner when $n = 3, 11$, as in the case of $n = 2$.

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